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AUTHORITY

AFATL ltr, 18 Nov 1975

AFATL-TR-72-28

AD894091

AIR ARMAMENT PLANNING AND DESIGN

THROUGH SYSTEMS ANALYSIS

PLANS OFFICE

TECHNIC   ORT AFATL-TR   28

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AIR FORCE ARMAMENT LABORATORY

AIR FORCE SYSTEMS COMMAND • UNITED STATES AIR FORCE

EGLIN AIR FORCE BASE, FLORIDA

Air Armament Planning and Design

Through Systems Analysis

John H. Arnold

FOREWORD

This report is based on a dissertation submitted to the graduate faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the Doctor of Philosophy in the Department of Mechanical, Aerospace and Industrial Engineering. The study was conducted between December 1970 and December 1971 and was not related to any specific Air Force project.

This technical report has been reviewed and is approved.

JOHN WARGO, JR., Lt Colonel, USAF
Chief, Plans Office

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ABSTRACT

Closed-form approximations for the probability of damaging surface targets with aerially delivered weapons are developed and analyzed for six different employment situations; single weapons against point targets, multiple weapons against point targets, single weapons against area targets, multiple weapons against area targets, single weapons against point targets with location uncertainty, and multiple weapons against point targets with location uncertainty. In each case, conditional damage and probability of coverage functions are developed, the product of which defines the probability of damage or probability of fractional damage depending upon whether the target is a point or an area, respectively. In addition, optimum damage probability constrained by specific design characteristics and delivery errors, is developed and compared with the capabilities of current systems. Optimum pattern radii or pattern radii which maximize the damage probability are also developed. Methodology which leads to preliminary design characteristics is developed through determination of the number of submunitions or weapon weight required to achieve any given level of damage for given employment constraints. Weapon preference methodology is developed which establishes a parametric evaluation procedure for weapon system employment preference and preliminary design characteristics. The methodology relates specifically to continuous patterns, that is, to weapons impact patterns bounded by singly connected curves and containing a random distribution of submunitions over the patterns. The principles are also extended to weapons systems with impact patterns that are annular in nature, either circular or elliptic (within established limits), and that are bound by multiply connected outer and inner curves. For this application, the submunitions are constrained to lie within the annular ring or the area between the outer and inner curves. The methodology is accurate and requires very little manpower and computer resources to employ. It is based on the mean area of effectiveness concept and can readily and accurately be used to assess the potential of new designs and proposals if accurate estimates of the mean area of effectiveness can be made from the lethal performance of existing munitions and submunitions.

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LIST OF ABBREVIATIONS AND SYMBOLS

Roman Symbol	Definition
A_C	Area of Target Covered
A_P	Single Weapon Pattern Area
A_V	Vulnerable Area of a Target
A_W	Multiple Weapon Pattern Area
CEA	Circular Error Average
CEP	Circular Error Probable
DEP	Deflection Error Probable
ECEP	Equivalent Circular Error Probable
F_C	Probability of Fractional Coverage
F_D	Probability of Fractional Damage
K	A Constant Related to Unitary Weapons
K_1	A Constant Related to Cluster Weapons
K_2	A Constant Related to Cluster Weapons
$\ln$	Natural or Napierian Logarithm
M	Number of Bomblets in a Massive Cluster
m	Number of Unitary Weapons in a Pattern of Area A
MAE	Mean Area of Effectiveness
MAE_B	Mean Area of Effectiveness of an Individual Bomblet
MAE_C	Mean Area of Effectiveness of a Cluster Weapon
MPI	Mean Point of Impact
N	Number of Cluster Weapons in a Pattern of Area A
n	Number of Bomblets in a Cluster Weapon

LIST OF ABBREVIATIONS AND SYMBOLS (Continued)

Roman Symbol	Definition
P_C	Probability of Covering a Point Target
P_D	Probability of Damaging a Point Target
$P_{D/C}$	Conditional Damage Given Coverage
P.E.	Probable Error
P_L	Pattern Length
P_W	Pattern Width
REP	Range Error Probable
R_L	Lethal Radius of a Unitary Weapon
R_{LB}	Lethal Radius of an Individual Bomblet
R.M.S.	Root-Mean-Square
R_P	Radius of a Cluster Weapon Pattern
R_T	Radius of an Area Target
R_V	Vulnerable Radius of a Target
R_W	Radius of a Multiple Weapons Pattern
r_1	Bomblet Reliability in a Cluster Weapon
W_B	Weight of an Individual Bomblet
W_{Bi}	Weight of the ith Bomblet
W_C	Weight of a Cluster Weapon
W_{CT}	Total Weight of N Multiply Delivered Cluster Weapons
W_{MC}	Weight of a Massive Cluster
W_U	Weight of a Unitary Weapon
W_{UT}	Total Weight of m Multiply Delivered Unitary Weapons

LIST OF ABBREVIATIONS AND SYMBOLS (Concluded)

Greek Symbols	Definition
α	A Constant Related to Unitary Weapons
β	A Constant of Proportionality Between Cluster Weapon Weight and Unitary Weapon Weight
μ	Mean Value
μ_X	Mean Value in Deflection
μ_Y	Mean Value in Range
π	3.14159
ρ	Correlation Coefficient
σ	Standard Deviation in Delivery Error
σ_A	Standard Deviation in Aiming Error
σ_B	Standard Deviation in Ballistic Error
σ_T	Standard Deviation in Total Error
σ_X	Standard Deviation in Deflection Error
σ_Y	Standard Deviation in Range Error
σ_1	Standard Deviation in Target Location Error

SECTION I

INTRODUCTION TO THE PROBLEM OF PLANNING AND DESIGNING
AIR ARMAMENT

A. Introductory Remarks

The purpose of this investigation is to develop closed form
approximations for the conditional damage and the probability of
coverage functions, products of which yield weapon system probability
of damage functions.   Closed form solutions in terms of the various
weapon, target and employment parameters, can be used both as a rapid
means of accurately assessing the effectiveness of weapons systems
and as a preliminary design tool for determining weapon systems de-
sign characteristics.

At present, the assessment of weapon system damage probability
is accomplished by numerical integration techniques which are time
consuming and require large expenditures of manpower and computer
resources[1,2]. Duncan[3] suggested the use of the Poisson distribution
as a means of approximating the hit probability of at least one missile
from a random circular salvo of missiles. Although narrow in scope,
it was this initial work which prompted the modifications and ex-
panded applications herein. Utilization of the Poisson distribution
for approximating the conditional damage function appears in reference
4. However, the probability of coverage function appears in functional
form such that the probability of damage must still be computed nu-
merically or found parametrically.

1,2 Superscripts are used to denote references

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Efforts to date have failed to produce sufficiently accurate and general closed form approximations of both the conditional damage and probability of coverage functions to serve as useful assessment and design tools.

The problem of planning and designing air armament is extremely complex, involving a chain of decision junctions many branches of which can lead to erroneous conclusions. Preliminary design is an essential element of air armament and development planning, the decision making process of weapon system selection. An important but often neglected consideration in the decision making process is whether one system is adequate or whether more than one system is required to maintain a realistic conventional munitions inventory capable of negating existing and anticipated threats. A basic question is whether the members of a family of weapon systems complement one another sufficiently to justify the additional expenditure that multiple systems imply. [4]

When several measures of effectiveness that are basically different may all be important for any given set of circumstances, serious consideration should be given to more than one system. In addition, if the expected employment environments are quite different, due either to future uncertainty or enemy counteraction, more than one system could yield the flexibility of action that is essential to perform at any reasonable level of competance. A single rigid weapon system invites enemy change which may negate the value of an otherwise effective system. [4]

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Very often "analysis" situations lead to the adoption of weapon systems which will never be called upon to operate in an average environment with respect to an average measure of effectiveness. These highly specialized weapon systems are expensive luxuries and can find justification only after a basic core of effective and highly flexible general purpose systems have become a reality. The development and maintenance of large numbers of highly specialized single purpose systems is prohibitively costly for the amount of anticipated return.

The development engineer is often too close to his programs to render rational and bias free assessments pertaining to current needs and future potential. As a consequence, it is essential that a periodic review of extant exploratory, advanced and engineering development programs be made by research and development management to ascertain the viability and currency of the existing programs with existing and anticipated levels of threat and tactical operational environments.

Until a few years ago, research and development of conventional munitions within the Air Force relied almost totally on the experience and judgment of development engineers. Although the basic analytical tools for weapons effectiveness analysis had been in existence for many years, they were used primarily as a means of assessing the effectiveness of the munition after it had been developed, tested and placed into inventory. Much of this was due to the fact that the necessary a priori inputs to a system analysis were scanty, unreliable and in many cases nonexistant. Very few development programs were actually justified based on predicted performance or anticipated payoff other than the "feelings" of development engineers.

More recent emphasis on pre-development analysis, promoted mainly by a tightening research and development budget and an ever expanding weapons systems inventory, has led to the establishment of more comprehensive physical testing and of a broad weapon effectiveness data bank. As a result, sufficient target vulnerability, weapons characteristics and weapons effects data have evolved which permit sound pre-development elimination of poor designs or retention of the most promising programs.

Current analytic efforts are extremely complex and weapon system analysis requires an expenditure of manpower comparable to that of the actual research and development of the weapon system. Current efforts involve extreme amounts of available computer resources. It is the nature of existing brute force techniques that discrimination between poor design and promising design can be made only after exhaustive computer studies.[1,2] In the final analysis the choice is still dependent upon the integrity of the analyst to ascertain the validity of the input data, the viability of the employment constraints utilized in the analysis, and the interpretation of the study results.

A major deficiency with current analytical efforts is the fact that poor systems are subjected to the same detailed scrutiny as the promising systems since discrimination can be made only in the final analysis. Analyses are conducted cautiously on all systems, promoted basically by a desire in the end to be able to discriminate between the promising systems and not between the two extremes. As a result, highly sophisticated system analysis techniques have been developed in an attempt to converge to true solutions with relatively small error.[1,2]

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Such techniques when applied to all proposed designs and concepts, result in a needless waste of resources. Techniques which are simple and require a minimum of manpower and computer resources are needed to reduce the ever increasing number of concepts and designs to the few which offer the greatest potential. The remaining programs can then be subjected to the detailed analysis necessary for establishing a viable research and development program.

It is the purpose of this effort to develop methodology which approximates with sufficient accuracy the potential of proposed weapons systems concepts and designs to the extent that the above can be realized.

B. Pattern for Employment of Tactical Air Forces [5]

The order of precedence in which combat air functions are accomplished cannot be prescribed by arbitrary methods and procedures. The fundamental principle governing the priority of combat air functions is the requirement to neutralize the enemy threat having the most profound and continued influence on the total mission of the combat area command. This principle is compatible with the inherent characteristics of tactical air forces, since it provides for their employment at a decisive time and place.

Tactical air forces are employed in the following tasks which produce area effects: the attainment of air superiority by destruction or neutralization of enemy air forces which threaten the area; the progressive neutralization of the enemy strength to sustain combat by isolating air and surface combat forces from their means of supply

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and battle sustenance; the disruption of enemy actions in the immediate area of engagement between the opposing surface forces. The priority of these tasks is dependent upon the effects desired in terms of the area command mission and war strategy.

Timely, offensive action against well-chosen targets is fundamental to the full exploitation of the combat potential of tactical air power. Timing of the action to destroy or neutralize a target may often be as important as the selection of the correct target.

The research and analysis of this effort will be restricted to tactical surface targets, specifically to the preliminary design and development planning methodology necessary to ascertain that a weapons system inventory is developed and maintained to meet all possible contingencies within this mission area.

C. Target Vulnerability[5]

The determination of target vulnerability involves an analysis of a number of complex factors. Premature commitment of forces without proper consideration of target vulnerability may result in needless expenditure of effort and resources with little appreciable effect upon the enemy's ability to conduct combat operations.

A fundamental factor in considering target vulnerability is the essentiality of the target to the enemy's combat effort. The breadth of this factor lies in an examination of the entire spectrum of targets within an area of operations. Pursuant to this examination, the targets chosen should involve the most significant areas of enemy strength, without which his combat operations may need to be reduced drastically or suspended entirely.

In order to determine the susceptibility of targets to destruction or neutralization, detailed knowledge of their physical features, such as mobility, mass, construction, location, and density is required. The vulnerability of a target or target system must then be measured against existing weapons to produce varying degrees of effect depending upon the specific tactical operational environment. If voids or marginal capabilities exist, new weapons systems must be planned, designed and placed at the disposal of the tactical air forces. A detailed understanding of weapon capabilities, limitations, and effects is required in order to make the most efficient and economical selection. Targets may be vulnerable to attack, but impervious to the weapons available at the time required. Capabilities must be sufficient to insure that the effects produced are commensurate with the effort and resources expended.

D.   Concepts From Weapons Systems Analysis

    1.   Damage and Casualty Criteria[6]

Because of the complexity of tactical air operations against surface targets, the changing nature of order of priority from one battle area to another, logistics, and a nonlinear depletion of available local munitions stockpiles; it has been necessary to assess the effect of weapons systems at various levels below the ultimate of catastrophic target damage. Thus, a number of personnel incapacitation criteria have evolved which relate weapon effectiveness at various levels below inducing death. They are separated into three main categories, with a time dependency within each category. The most stringent category is incapacitation to the extent that personnel

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can no longer defend themselves from attack. The second category

denies personnel the ability to assault, and the least stringent

criterion denies the ability to function in a supply effort.

Effects on material targets are also assessed at levels of

damage which are less than catastrophic. If a target possesses

mobility, a group of time dependent criteria relating to immobiliza-

tion has evolved and, if a target possesses firepower, criteria

have evolved which relate the potential of denying the target its

firepower.

The interpretation of "kill" probabilities requires an under-

standing of some of the basic principles underlying weapon systems

analysis. The term "kill" does not necessarily mean a kill in the

literal sense. It is defined in terms of the desired degree of damage

insofar as material targets are concerned and in terms of level of

incapacitation insofar as personnel targets are concerned. These

criteria have physical and not statistical interpretations.[6]

Each weapon has a certain probability of defeating any target

(including the null probability) for any assigned damage or casualty

criterion. Some of the more commonly used criterial are listed in

Table I.

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TABLE I.

SELECTED DAMAGE AND CASUALTY CRITERIA[6]

Target	Criterion	Definition
Truck	A Kill	Vehicle Stoppage Within 2 Minutes
	B Kill	Vehicle Stoppage Within 20 Minutes
Tank	F Kill	Complete or Partial Loss of Fire-power
	K Kill	Castastrophic Damage
	M Kill	Immediate Immobilization
	A Kill	Immobilization Within 2 Minutes
Aircraft	B Kill	Control Loss Within 5 Minutes
	C Kill	Mission Abort
	K Kill	Control Loss Within 5 Seconds Resulting in Eventual Catastrophic Damage
	KK Kill	Immediate Catastrophic Damage
Bridge	S1	Drop a Single Span
Rail	Cut	Cut a Single Rail
Personnel	30 sec Def.	Ability to function in a defensive posture denied within 30 seconds
	5 min Ass.	Ability to function in an assault role denied within 5 minutes

2. Mean Area of Effectiveness (MAE)[6]

The MAE concept relates the effectiveness of a weapon against a particular target, about the target centroid, in terms of the weapon's characteristics; target characteristics, both physical and vulnerable; and a specified damage or incapacitation criterion. It has been developed and can be defined such that if the target is located within the mean area of effectiveness for a specified weapon and damage or incapacitation criterion, then the damage or incapacitation criterion is at least satisfied.

This definition applies directly to point targets but may be extended to include all targets whose centroids are located within the MAE. Certain targets having edge effects (unitary targets having large distributed areas such as buildings) and of a class where partial damage assessment has military significance must be approached in an entirely different manner. The reader is referred to reference 6 for further explanation.

The problem of assessing weapon system effectiveness is essentially reduced to determining the probability that a target will lie within the mean area of effectiveness subject to the constraints imposed by the weapon delivery system. The MAE concept may be modified, without loss of generality, for multiple weapons delivery, as will be shown in later developments.

3. Delivery Accuracy

Delivery accuracy with regard to current capabilities is a misnomer, since in the most basic sense, both the standard deviation (σ) and circular probable error are measures of the pilot's inability

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to place his weapon or weapon pattern center on a desired point on the ground. Combat and testing experience has shown that errors in range and deflection can usually be described by a normal (or a Gaussian) distribution whose precise characteristics are well known[6,7]. The range error probable (REP) and deflection error probable (DEP) measure the tendency of the impact points to differ from the target center. When the REP and DEP are identical, the resulting distribution is called circular or radial. Circular distributions can further be described in terms of the circular probable error (CEP).[7]. When REP and DEP are not equal, the CEP concept may still be employed as, related in paragraph (E.3) of this section.

Delivery accuracy is normally divided into two categories, one being referred to as aiming error (σ_A) and the other as ballistic error (σ_B). The aiming error is attributed basically to the pilot and his ability or inability to place the mean point of impact (MPI) of the weapon system on the target center. The magnitude of this error depends significantly on pilot experience and initiative, the physical and defensive environment, and the handling qualities of the aircraft. The ballistic errors are attributed to the weapon and are basically a measure of the divergence of impacts within a weapons pattern with respect to the MPI. The magnitude of the errors depends significantly upon the quality control in the weapons production and the ejection system.

Conversions for REP, DEP, CEP and σ are given in Table II. For ease of tabulation, both REP and DEP are referred to as the probable error (PE).

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TABLE II.			
DELIVERY ACCURACY CONVERSIONS [8]			
To Convert From	To	Multiply	By
CEP	σ	CEP	0.8493
CEP	PE	CEP	0.5729
σ	PE	σ	0.6745
σ	CEP	σ	1.1774
PE	σ	PE	1.4826
PE	CEP	PE	1.7456

4.   Damage Probability

In the most general sense, the probability of damage (P_D) is given by:[1]

$$P_D = \iiint_{-\infty}^{\infty} p(x,y,z)f(x,y,z) \ dx \ dy \ dz \qquad (1\text{-}1)$$

where,

$p(x,y,z)$ is the kill probability of a warhead detonating at (x,y,z) and,

$f(x,y,z)$ is the probability density function for the warhead detonating at (x,y,z).

A closed form solution to this function does not exist and it must be numerically integrated at discrete points in a manner so as to converge to the optimal parameter values as efficiently as possible.

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An alternative expression for the damage function is:

$$P_D = P_{D/C} \cdot P_C \qquad (1\text{-}2)$$

where $P_{D/C}$ is the conditional damage given coverage and P_C is the probability of converage.

Accurate closed form solutions for the damage probability are possible with this approach through utilization of the MAE concept and provided the conditional damage and probability of coverage functions exist. It is this approach which will be followed throughout this investigation.

For area targets, the form of the probability of fractional damage is the same and is given by:

$$F_D = P_{D/C} \cdot F_C \qquad (1\text{-}3)$$

where $P_{D/C}$ is conditional damage given coverage and F_C is the fraction of the target covered.

E.   Concepts from Statistics

   1. Poisson Distribution

      The discrete probability distribution

$$p(k) = \frac{\lambda^k e^{-\lambda}}{k!} \; ; \; k = 1,2,3,\ldots; \; p(k) = 0 \text{ otherwise} \qquad (1\text{-}4)$$

is called the Poisson distribution after Poisson who developed it in

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the early part of the 19th century. The distribution has the following properties:

Mean	$\mu = \lambda$
Variance	$\sigma^2 = \lambda$
Standard Deviation	$\sigma = \sqrt{\lambda}$

2.   Bivariate Normal Distribution[8,9]

The two dimensional bivariate normal population is the probability space induced by a pair of random variables (x,y) having a joint density function given by:

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \; \exp\left\{- \frac{1}{(1-\rho^2)} \left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 -2\rho \left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2 \right]\right\} \tag{1-5}$$

Carrying out the required integration the marginal (Gaussian) density function $f(x)$ of x is: [9]

$$f(x) = \frac{1}{\sqrt{2\pi}\;\sigma_x} \; \exp\left[- \frac{1}{2} \left(\frac{x-\mu_x}{\sigma_x}\right)^2\right] \tag{1-6}$$

and $g(y)$ of y is:

$$g(y) = \frac{1}{\sqrt{2\pi}\;\sigma_y} \; \exp\left[- \frac{1}{2} \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right] \tag{1-7}$$

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Thus x and y are normally distributed random variables with means μ_x and μ_y and standard deviations σ_x and σ_y.

The expectation

$$E\left[\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right)\right] \tag{1-8}$$

is the constant ρ, the correlation coefficient of the random variables x and y ($0 \leq \rho \leq 1$). If the correlation coefficient has an absolute value of unity, the joint density function is meaningless, and the variables x and y are said to have a singular normal distribution, the entire probability mass being concentrated on a line. There is complete linear dependence between x and y for this case.

If $\rho = 0$, x and y are uncorrelated, hence independenc, and the joint density function is a product of the marginal density functions.[9]

$$f(x,y) = f(x)g(y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left\{-\frac{1}{2}\left[\left(\frac{x-\mu_y}{\sigma_x}\right)^2 + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right\} \tag{1-9}$$

If in the bivariate normal density function above $\sigma_x = \sigma_y = \sigma$, the distribution is said to be circular normal. For mean values $\mu_x = \mu_y = 0$ the function is normalized:[9]

$$f(x,y) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{1}{2}\left(\frac{x^2 + y^2}{\sigma^2}\right)\right] \tag{1-10}$$

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In terms of the vector deviations from the mean value, when only the magnitude of the radial error is significant, the density function is:[9]

$$f(r) = \frac{r}{\sigma^2} \exp\left[-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2\right] \tag{1-11}$$

This function is often referred to as the radial or Rayleigh density function. These distributions have the following properties:[8]

a) Marginal Density Functions   (Figure 1)

    Standard Deviations σ_x, σ_y

    Probable Error in x $.67449\,\sigma_x$

    Probable Error in y $.67449\,\sigma_y$

b) Joint Density Function ($\sigma_x = \sigma_y$) (Figure 1)

    Standard Deviation $\gamma = \sqrt{2}\,\sigma$

    Circular Probable Error $CEP = \sqrt{2 \ln 2}\,\sigma$

    Circular Error Average $CEA = \sqrt{\pi/2}\,\sigma$

3.   Equivalent Circular Probable Error

The circular probable error, as a parameter, is a unique function of the circular normal distribution. Although it is not associated with the non-circular (elliptic) bivariate normal distribution, there is a circle centered at the aiming point of the non-circular distribution which contains half of the sample points. The radius of this circle is often referred to as equivalent circular probable error (ECEP).[7]
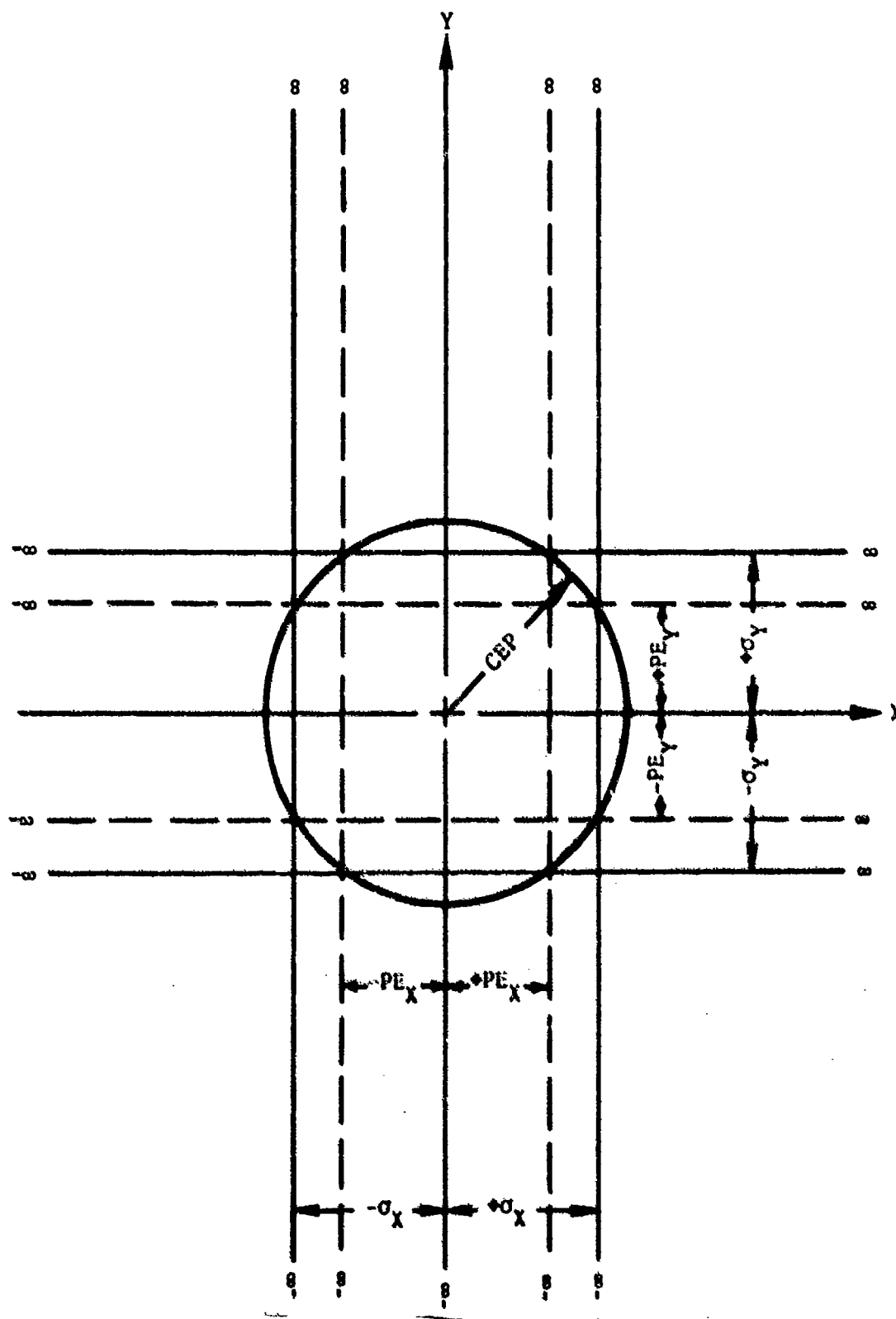
Figure 1. Circular Normal Distribution

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Radial distributions based on an elliptical normal distribution ($\sigma_x \neq \sigma_y$) must be integrated numerically for accurate results. However, when the smaller distribution is at least one-third of the larger ($.33 \leq \sigma_x/\sigma_y \leq 3.0$), good approximations can be obtained by a straight line fit to the exact function:

$$\text{ECEP} = 0.615\,\sigma_y + 0.562\,\sigma_x \qquad (\sigma_y \leq \sigma_x) \qquad (1\text{-}12)$$

$$\text{ECEP} = 0.615\,\sigma_x + 0.562\,\sigma_y \qquad (\sigma_x \leq \sigma_y) \qquad (1\text{-}13)$$

Other common approximations used include: the geometric mean $\sqrt{\sigma_x \sigma_y}$, which is accurate for very low values of cumulative probability and is reasonably accurate up to a cumulative probability of 0.4; the arithmetic mean $\left[\left(\sigma_x + \sigma_y\right)/2\right]$ which is excellent at 0.6 and is often used for intermediate values including the 50% point (CEP); and, the root-mean-square $\left[\left(\sigma_x^2 + \sigma_y^2\right)/2\right]^{1/2}$, a good approximation above a cumulative probability of 0.75[8]. The best overall approximation is the curve fit since the interval of accuracy extends over and beyond the other three. These approximations in terms of the dimensionless parameters (ECEP/σ_x; σ_y/σ_x) and (ECEP/σ_y; σ_x/σ_y) are shown in Figure 2.

Figure 2. Equivalent Circular Error Probable[7]
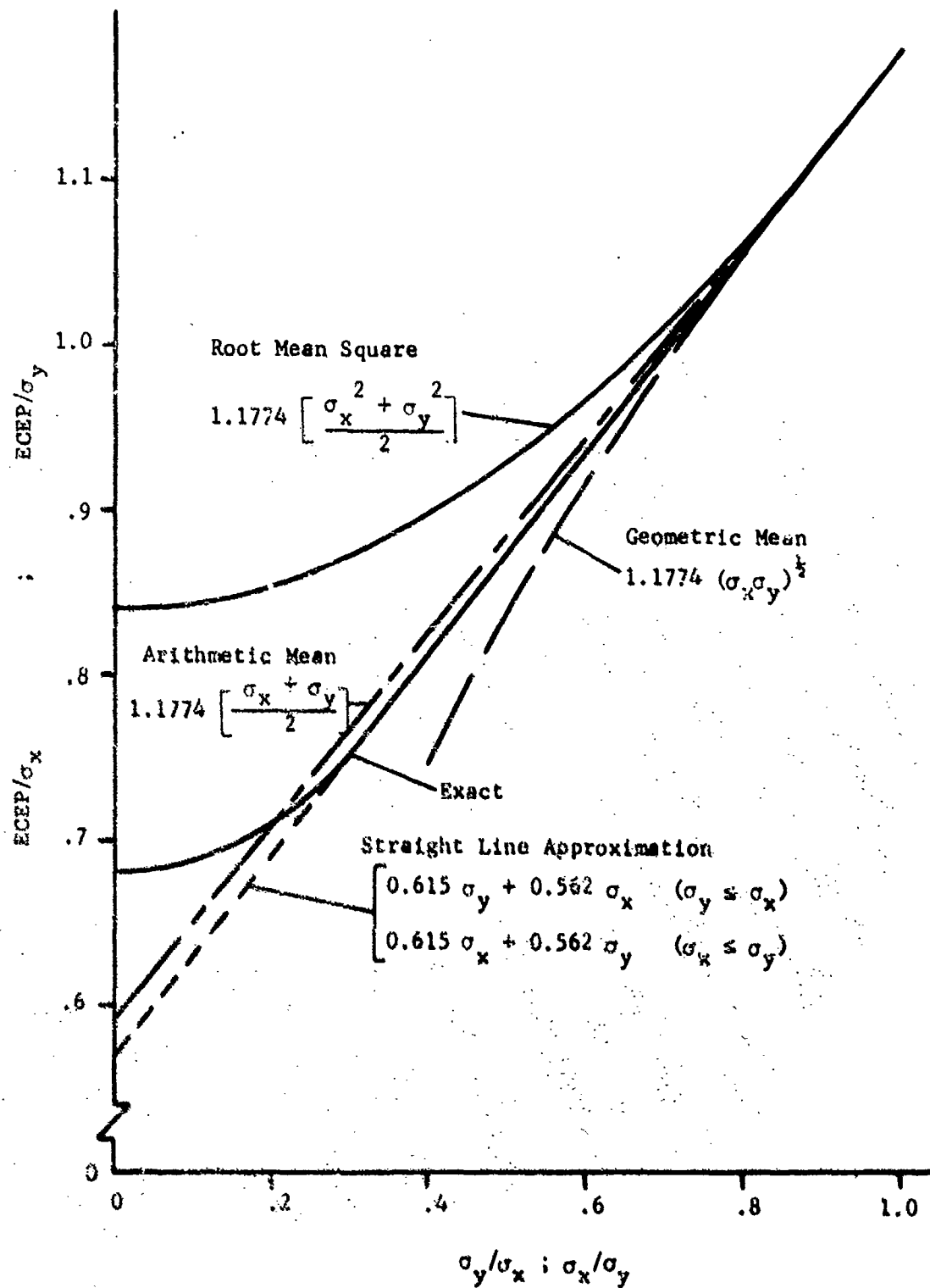
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OPTIMUM DAMAGE PROBABILITY AGAINST POINT TARGETS

A.   Singly Delivered Weapons Against a Point Target

   1.   The Probability of Coverage, Conditional Damage and Probability

        of Damage Functions for Area Weapons.

        This section contains the development of the damage equations

for an area weapon delivered against a point target. An area weapon

is defined as a weapon system which contains a number (n) of submuni-

tions which are released in a salvo (simultaneously). In most cases,

a cluster (packaged submunitions) is released as a unit and at some

point along its trajectory dispenses the submunitions in a salvo.

A point target is defined as a single target containing one or more

vulnerable components, any of which or any combination of which, satis-

fies the damage criterion if rendered inoperable.

        a.   The Probability of Coverage Function

        The probability of coverage function P_C can be approximated

by the radial distribution. It has been shown in reference 6, and is

briefly discussed in the introduction, that the circular normal or

radial distribution function is an excellent approximation of the

elliptic normal distribution for $.33 \leq \sigma_x/\sigma_y \leq 3.0$. In these cases

it is generally more appropriate to convert from standard deviation to

circular probable error (CEP).

        For the case of a point target ($R_T \ll R_p$) centered at (0,0), the

probability that the target centroid will lie within the weapon's pattern

radius R_p is the density function integrated over R_p. It can be assumed

that the pilot can identify and is delivering weapons to the target

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center with standard deviation error σ. Thus the expected position of the munition's pattern center is the target centroid.

$$P(R_p) = \int_0^{R_p} \left[\frac{r}{\sigma^2} \exp(-r^2/2\sigma^2) \right] dr \qquad (2\text{-}1)$$

$$P_C = P(R_p) = \left[1 - \exp(-R_p^2/2\sigma^2) \right] \qquad (2\text{-}2)$$

This is a special case of the probability of covering an area target with an area munition with the area target radius degenerating to zero. The development of the more general problem will be covered in Section III. The point target problem is separated from the general problem primarily due to the higher frequency with which the former appears in relation to the latter. Approximately 80% of all aerially attacked surface targets can be classed as point targets.

Equation (2-2) is not necessarily restricted to point target applications. As a matter of fact, it is an excellent approximation for most area targets of interest in tactical conventional and counter-insurgency warfare.[10] For target radii to standard deviation ratios on the interval $(0 \le R_T/\sigma \le 0.5)$, Equation (2-2) accurately describes the circular coverage function[10] over the entire range of $R_p (0 \le R_p/\sigma \le \infty)$. Restrictions on the latter interval increase the range of R_T/σ. For example, for $R_p/\sigma \gtrsim 3.0$, the function is accurate over the interval $(0 \le R_T/\sigma \le 1.5)$, for $R_p/\sigma \gtrsim 4.0$, it is extended over the interval $(0 \le R_T/\sigma \le 3.0)$, and for $R_p/\sigma \gtrsim 5.0$, it applies to $(0 \le R_T/\sigma \le 4.0)$.

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Even for the best contemporary combat delivery accuracies, the ratios R_P/σ and R_T/σ will not normally exceed 5.0. However, the advent of improved delivery accuracy necessitates an extension of the principles over larger intervals. As the standard deviation of error is improved, the range on R_T must be reduced proportionately to maintain the specified intervals of R_T/σ. Developments in the next section will relate the extension over the interval $(0 \leq R_T/\sigma \leq \infty)$ for all $R_P(0 \leq R_P/\sigma \leq \infty)$.

b. The Conditional Damage Function

For weapons systems having a random uniform distribution of bomblets within the pattern area (A_P) bounded by a singly connected curve, the Poisson approximation adequately represents the conditional damage probability $(P_{D/C})$. The conditional damage probability is defined as the average damage over the weapon pattern (Figure 3). Let

$$p(k \text{ successes}) = \exp (-\mu) \, \mu^k/k! \tag{2-3}$$

where

$$\mu = r_1 n \ \text{MAE}_B/A_P = r_1 n \ R_{LB}{}^2/R_P{}^2 \qquad (R_{LB} \ll R_P)$$

The mean value μ is the ratio of the total area within which the damage criterion is satisfied to the total pattern area. The total area of effectiveness is the product of the number of bomblets with the system and the individual bomblet MAE. Therefore,

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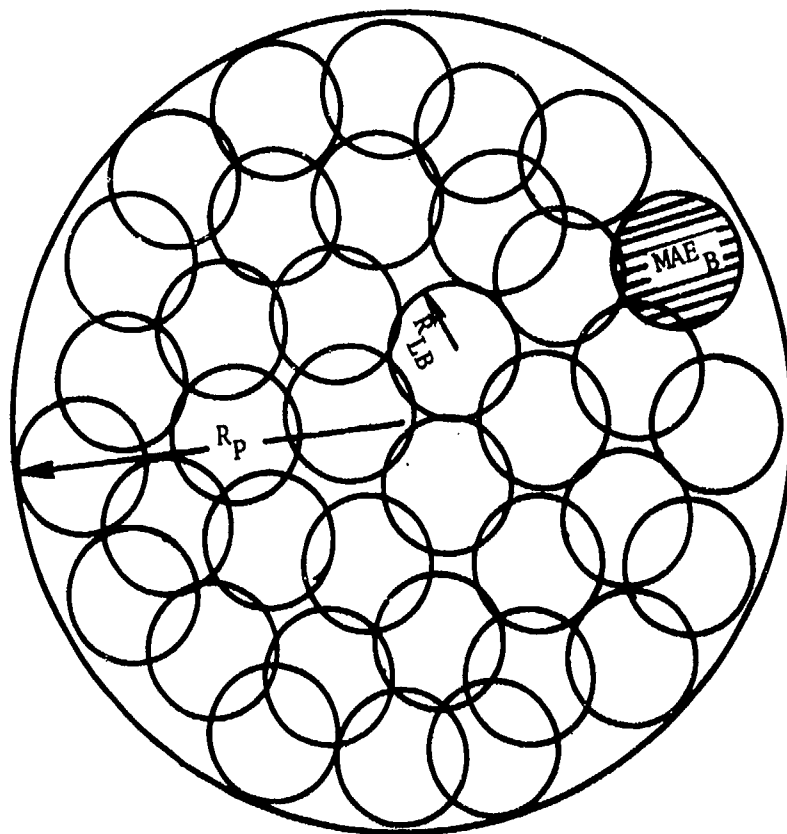
$$\mu = \sum_{i=1}^{n} r_1 MAE_{B_i} / \pi R_p^2 = r_1 n R_{LB}^2 / R_p^2$$

$$P_{D/C} = 1 - \exp{(-r_1 n R_{LB}^2 / R_p^2)}$$

Figure 3. Conditional Damage for a Single Area Weapon

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$$P_{D/C} = 1 - p(0\ \text{Successes}) = 1 - \exp\,(-\mu)\ \mu^{0}/0!$$

or

$$P_{D/C} = \left[1 - \exp\,(-r_1 n\ R_{LB}{}^{2}/R_P{}^{2}) \right] \qquad (2\text{-}4)$$

    c. The Probability of Damage Function

    The product of Equations (2-1) and (2-3) leads to:

$$P_D = \left[1 - \exp\,(-r_1 n\ R_{LB}{}^{2}/R_P{}^{2}) \right] \left[1 - \exp\,(-R_P{}^{2}/2\sigma^{2}) \right] \qquad (2\text{-}5)$$

    2. The Probability of Damage Function for Unitary Weapons.

Unitary weapons are divided into two separate groups for class-ification but can be treated identically in the development of the damage function. One class of unitary weapons is characterized by high-explosive/blast/fragmentation warheads for which the MAE concept is defined. Another class is comprised of kinetic energy and shaped-charge penetration warheads, for which the vulnerable area (A_V) concept is defined. For unitary weapons characterized by the MAE concept, the conditional damage probability is unity. The MAE is defined in such a manner that, given the target centroid is located within the MAE, the damage criterion is satisfied, thus reducing the problem to coverage expectancy. The vulnerable area concept is defined such that, given an impact within the vulnerable area, the damage criterion is sat-isfied thereby reducing this problem to one of hit expectancy. Thus,

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the two concepts are equivalent. Since in the case of unitary warheads, $MAE = A_p$, the damage probability based on the Rayleigh distribution becomes:

$$P_D = \left[1 - \exp \left(-MAE/2\pi\ \sigma^2 \right) \right] \tag{2-6}$$

and for the vulnerable area concept:

$$P_D = \left[1 - \exp \left(-A_V/2\pi\ \sigma^2 \right) \right] \tag{2-7}$$

and since $MAE = \pi R_L^2$ and $A_V = \pi R_V^2$,

$$P_D = \left[1 - \exp \left(-R_L^2/2\sigma^2 \right) \right] \tag{2-8}$$

and

$$P_D = \left[1 - \exp \left(-R_V^2/2\sigma^2 \right) \right] \tag{2-9}$$

3. Maximization of the Damage Functions

The damage function in Equation (2-5) may be maximized to:

$$P_D = \left[1 - \exp \left(-R_p^2/2\sigma^2 \right) \right] \tag{2-10}$$

simply by increasing the number of bomblets within the weapon without bound, or both. None of these alternatives is economically feasible and even if such was the case, the limiting case (2-10) becomes

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analogous to the already limiting case for unitary weapons as reflected in Equations (2-8) and (2-9). Within practical employment and economic constraints, the damage function may be maximized for a fixed munitions design concept by controlling pattern size as a function of the standard deviation of error (σ), the number of submunitions (n) and the MAE_B of the individual submunitions. This approach is practical since pattern size is a unique function of the aircraft weapon release parameters and weapon (fuze) function altitude. Thus, for a given standard deviation or error, a fixed number of submunitions each having a mean area of effectiveness characterized by (R_{LB}), the damage function may be maximized by determining the optimum pattern radius for the above constraints. The optimum pattern radius may be achieved by specifying aircraft release parameters and a fuze function altitude (altitude at which the cluster releases the submunitions) which may be preset or electronically set from the cockpit.

As previously related, the damage function may be specified by:

$$P_D = P_{D/C} \cdot P_C$$

Operating on the damage function

$$\frac{\partial P_D}{\partial R_p} = P_{D/C} \frac{\partial P_C}{\partial R_p} + P_C \frac{\partial P_{D/C}}{\partial R_p} \qquad (2-11)$$

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Equating dP_D/dR_P to zero and solving for

$$\frac{P_C}{P_{D/C}} = - \frac{\partial P_C/\partial R_P}{\partial P_{D/C}/\partial R_P} \qquad (2\text{-}12)$$

Then in terms of Equation (2-5),

$$\frac{\partial P_C}{\partial R_P} = \frac{R_P}{\sigma^2} \exp\left[-R_P^2/2\sigma^2\right] \qquad (2\text{-}13)$$

$$\frac{\partial P_{D/C}}{\partial R_P} = - \frac{2r_1 n R_{LB}^2}{R_P^3} \exp\left[-r_1 n R_{LB}^2/R_P^2\right] \qquad (2\text{-}14)$$

Substituting these relations and the expressions for P_C and $P_{D/C}$ into (2-12) yields:

$$\frac{\left[1 - \exp\left(-R_P^2/2\sigma^2\right)\right]}{\left[1 - \exp\left(-r_1 n R_{LB}^2/R_P^2\right)\right]} = \frac{R_P^4 \exp\left[-R_P^2/2\sigma^2\right]}{2r_1 n R_{LB}^2 \sigma^2 \exp\left[-r_1 n R_{LB}^2/R_P^2\right]} \qquad (2\text{-}15)$$

Equation (2-15) is satisfied when:

$$R_P^4 = 2r_1 n R_{LB}^2 \sigma^2 \qquad (2\text{-}16)$$

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The pattern radius which maximizes the damage function is:

$$R_P = (2r_1 n R_{LB}^2 \sigma^2)^{1/4} \tag{2-17}$$

Substituting this expression into Equation (2-5) and simplifying, yields the damage function for the optimum pattern radius:

$$P_D = \left[1 - \exp \left(-R_{LB} \sqrt{r_1 n} \Big/ \sqrt{2}\, \sigma \right) \right]^2 \tag{2-18}$$

In many instances, it is desirable to determine the weapon system characteristics which will yield a desired level of damage (P_D) for any given submunition type and standard deviation of error σ. The number of bomblets required to obtain a level of damage P_D with standard deviation of aiming error σ can be determined from (2-18).

$$n = (2/r_1) \left[(\sigma/R_{LB}) \ln \left(1 - \sqrt{P_D} \right) \right]^2 \tag{2-19}$$

The number of submunitions n (of a given type) completely specifies the weight, volume, and physical characteristics of the weapon system required to yield the desired probability of damage for any given submunition design constrained by a standard deviation of error σ.

It is significant to note that the condition

$$R_P^4 = 2r_1 n R_{LB}^2 \sigma^2$$

is equivalent to the following expressions:

$$\frac{R_P^{\ 2}}{2\sigma^2} = \frac{r_1 n\, R_{LB}^{\ 2}}{R_P^{\ 2}}$$

This implies that the damage function is maximized when the conditional damage over the weapons pattern is equal to the probability of coverage $(P_{D/C} = P_C)$.

4. Optimum Cluster Weapons Versus Optimum Unitary Weapons

    a. Optimum Cluster in Terms of Circular Probable Error

        The circular probable error can be expressed in terms of the standard deviation of aiming error:

$$CEP = \sqrt{2\, \ell n\, 2}\ \sigma$$

or

$$\sigma = CEP / \sqrt{2\, \ell n\, 2}$$

substituting this expression into Equation (2-18)

$$P_D = \left[1 - \exp\left(-R_{LB} \sqrt{r_1 n\, \ell n\, 2} \big/ CEP\right) \right]^2 \tag{2-20}$$

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or, in terms of CEP

$$CEP^2 = \frac{r_1 n R_{LB}^2 \ln 2}{\left[\ln(1 - \sqrt{P_D})\right]^2}$$

(2-21)

b. Circular Probable Error in Terms of weapon Weight

It is desirable to compare the effectiveness of cluster weapons and unitary weapons on an equal weight basis or cost/weight basis since weapons weight is a primary penalty on aircraft performance in terms of acceleration, range and endurance. Cost may enter the problem since usually minimization of the penalty must be traded off against an increase in costs.

Define a constant K_1 such that:

$$K_1 = \frac{\text{Total Cluster Weight}}{\text{Total Bomblet Weight}} = \frac{W_c}{\displaystyle\sum_{i=1}^{n} W_{Bi}}$$

K_1 is the reciprocal of the packaging efficiency. Define a constant K_2 such that:

$$K_2 = W_B$$

Then, the number of bomblets in the cluster can be expressed as:

$$n = \sum_{i=1}^{n} W_{Bi}/W_B$$

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and the cluster weight as:

$$W_C = \left(W_C / \sum_{i=1}^{n} W_{Bi}\right)\left(W_{Bi}\right)n = K_1 K_2 n$$

or in terms of n,

$$n = W_C / K_1 K_2$$

Substituting this expression into Equation (2-21)

$$CEP^2 = \frac{r_1 \, W_C \, R_{LB}^2 \, \ell n \, 2}{K_1 K_2 \left[\ell n \, (1 - \sqrt{P_D})\right]^2} \; ; \quad (0 < P_D < 1) \tag{2-22}$$

c. Optimum Unitary in Terms of Circular Probable Error

The expression for σ can be substituted into Equation (2-6) yielding:

$$P_D = \left[1 - \exp \, (- \, MAE \, \ell n \, 2 / \pi \, CEP^2)\right] \tag{2-23}$$

In terms of CEP^2,

$$CEP^2 = - \frac{MAE \, \ell n \, 2}{\pi \, \ell n \, (1 - P_D)} \; ; \quad 0 < P_D < 1 \tag{2-24}$$

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d. Circular Probable Error in Terms of Unitary Weapon
    Weight

Examination of the behavior of MAE (Figure 4) as a
function of weapon weight for unitary weapons reveals a linear
logarithmic-logarithmic relationship for constant (or nearly constant)
charge mass to metal mass ratios (C/M). Of particular interest is
the family of general purpose unitary weapons (C/M $\approx$ 1.0) (Figure 4).

The mean area of effectiveness may be expressed as:

$$\ell n \; \text{MAE} = \ell n \; K + \alpha \; \ell n \; W_u$$

where K is the MAE of the smallest weight being considered (equivalent
to the MAE axis intercept), α is the slope of the line, and W_u is the
weight of the unitary weapon being evaluated.

Alternatively,

$$\text{MAE} = K \; W_u^{\alpha}$$

substituting this expression into Equation (2-24)

$$\text{CEP}^2 = -\frac{K \; W_u^{\alpha} \; \ell n \; 2}{\pi \; \ell n \; (1-P_D)} \quad ; \; \left| 0 < P_D < 1 \right| \tag{2-25}$$

e. Cluster Weight versus Unitary Weight

By equating Equations (2-25) and (2-24) the following
expression is obtained:

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1. Light Material
2. Light Material
3. Motor Vehicle (Target 1)
4. Prone Personnel (Target 2)
5. Motor Vehicle
6. Light Armor
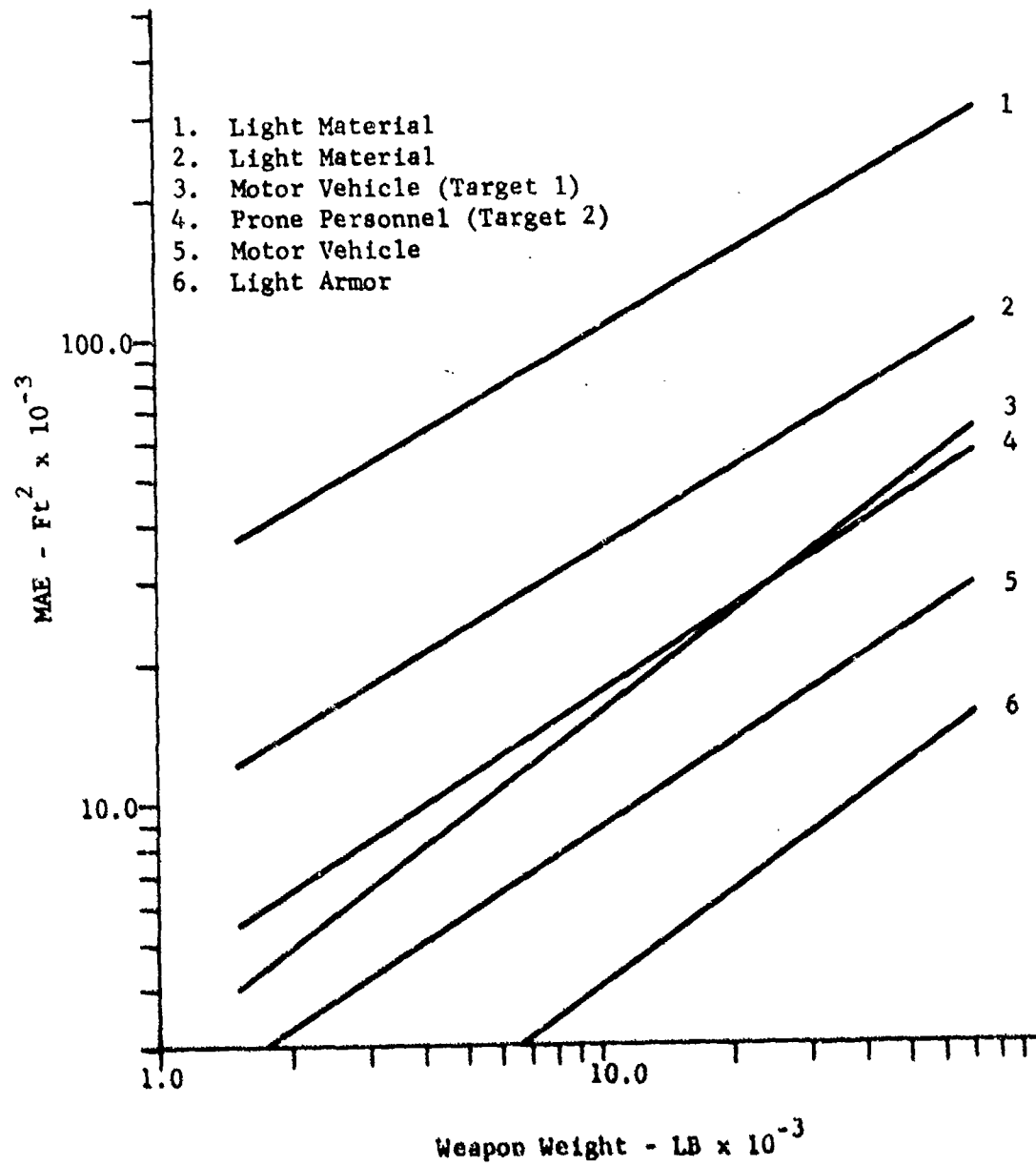
Figure 4. Typical Unitary MAE[7]

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$$\frac{r_1 \, W_C \, R_{LB}^{\,2} \, \ell n \, 2}{K_1 K_2 \left[\ell n \, (1 - \sqrt{P_D}) \right]^2} = \frac{K \, W_u^{\alpha} \, \ell n \, 2}{\pi \, \ell n \, (1 - P_D)} \; ; \; (0 < P_D < 1)$$

This approach permits the development of a weight comparison when both weapon systems are delivered with the same aiming error, or equivalently, from identical delivery aircraft.

Solving for W_C in terms of W_u^{α}

$$W_C = - \left(\frac{K_1 K_2 K}{\pi \, r_1 \, R_{LB}^{\,2}} \right) \left\{ \frac{\left[\ell n \, (1 - \sqrt{P_D}) \right]^2}{\ell n \, (1 - P_D)} \right\} W_u^{\alpha} \; ; \; (0 < P_D < 1) \qquad (2\text{-}26)$$

For any specified level of damage desired (P_D) and for equal weapon system weights $(W_C = W_u)$ where the weapon systems designs are specified:

$$\beta = - \left(\frac{K_1 K_2 K}{\pi \, r_1 \, R_{LB}^{\,2}} \right) \left\{ \frac{\left[\ell n \, (1 - \sqrt{P_D}) \right]^2}{\ell n \, (1 - P_D)} \right\} \; ; \; (0 < P_D < 1)$$

and Equation (2-26) becomes

$$W_C = \beta \, W_u^{\alpha}$$

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or

$$W = \beta \, W^{\alpha} = \beta^{\left(\frac{1}{1-\alpha}\right)}$$

and finally,

$$W = \left[- \left(\frac{K_1 K_2 K}{r_1 \, \pi \, R_{LB}^2}\right) \left\{ \frac{\left[\ln\left(1 - \sqrt{P_D}\right)\right]^2}{\ln\left(1 - P_D\right)} \right\} \right]^{\left(\frac{1}{1-\alpha}\right)} ; \quad (0 < P_D < 1) \qquad (2\text{-}27)$$

By employing the relationship for weight in the above expression with either Equation (2-22) and (2-25) and parametric values of P_D ($0 < P_D < 1.0$), weight and CEP; a curve is obtained which divides the CEP-weight plane into distinct areas where either cluster weapons or unitary weapons are preferred. In addition, Equations (2-22) and (2-25) will yield a family of constant P_D curves (parallel straight lines in the logarithmic CEP^2 vs weight plane) which may be utilized both as a weapon effectiveness tool or as a parametric design tool.

B. Multiply Delivered Weapons Against a Point Target

1. The Probability of Coverage, Conditional Damage and Probability of Damage Functions for Multiply Delivered Area Weapons

This section contains the development of the damage equations for multiply delivered area weapons against a point target. The development is an excellent approximation for multiple cluster patterns which are circular (Figure 1) and does not diverge severely from the numerically integrated solution for rectangular weapons patterns in

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the interval $(.33 \leq P_W/P_L \leq 3.0)$ where P_W is the pattern width and P_L is the pattern length. In this case, the pattern area $P_W P_L$ is approximated by πR_p^2.

a. The Probability of Coverage Function

The coverage function is identically the same form as defined in paragraph (II-A-1-a). However, in this case it is desirable to determine the probability that the target lies within the multiple area weapons pattern of radius R_W. The damage function will be developed later in terms of R_W.

In a procedure identical to the approach in paragraph (II-A-1-a) the coverage function can be developed. It becomes:

$$P_C{}' = \left[1 - \exp \left(- R_W^2/2 \, \sigma^2\right) \right] \qquad (2\text{-}28)$$

b. The Conditional Damage Function

The product of the conditional damage within a single cluster pattern and the pattern area is defined as the mean area of effectiveness of the cluster (Figure 5).

$$MAE_C = \pi R_p^2 \, P_{D/C}$$

The mean value can then be determined

$$\mu = N(MAE_C)/A_W = N R_p^2 \, P_{D/C}/R_W^2$$

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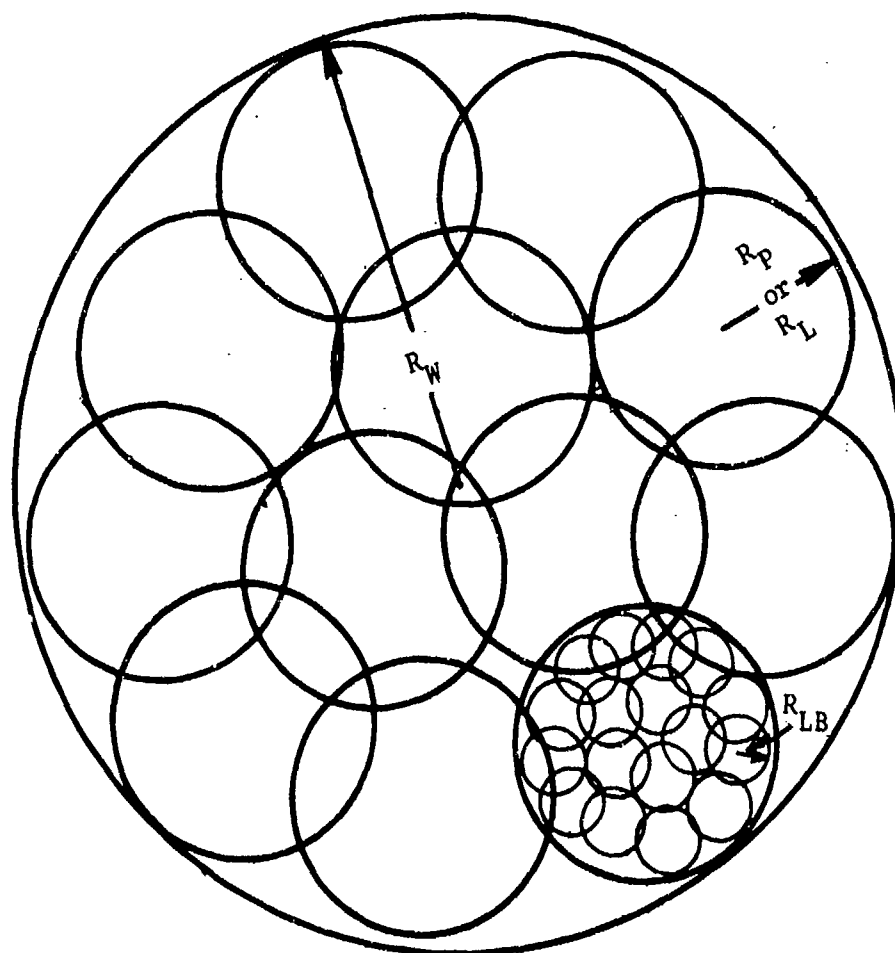
$$\mu = \sum_{i=1}^{N} \pi R_{pi}^{2} P_{D/C} / \pi R_{W}^{2} = N R_{p}^{2} P_{D/C} / R_{W}^{2} \quad \text{(Area)}$$

$$\mu = \sum_{i=1}^{m} \pi R_{Li}^{2} / \pi R_{W}^{2} = m R_{L}^{2} / R_{W}^{2} \quad \text{(Unitary)}$$

$$P_{D/C} = 1 - \exp \left(-N R_{p}^{2} P_{D/C} / R_{W}^{2}\right) \quad \text{(Area)}$$

$$P_{D/C} = 1 - \exp \left(-m R_{L}^{2} / R_{W}^{2}\right) \quad \text{(Unitary)}$$

Figure 5. Conditional Damage for Multiply Delivered Weapons

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It should be noted that the mean value in this sense is not the same as would be derived for the case of the product Nn bomblets distributed uniformly over A_W determined by

$$\mu = r_1 \, Nn \, R_{LB}^2 / R_W^2$$

When area weapons are delivered multiply, n bomblets per pattern are constrained to lie in N pattern areas, the N patterns distributed randomly over A_W. The two terms converge only for N coincident patterns of radius $R_p = R_W$.

Therefore, the conditional damage for multiply delivered area weapons becomes:

$$P'_{D/C} = \left[1 - \exp\left(- NR_p^2 \, P_{D/C}/R_W^2\right) \right] \tag{2-29}$$

where R_p is determined by Equation (2-17) and $P_{D/C}$ by Equation (2-4).

c. The Probability of Damage Function

The probability of damage is determined by the product of Equations (2-28) and (2-29)

$$P'_D = \left[1 - \exp\left(- NR_p^2 \, P_{D/C}/R_W^2\right) \right] \left[1 - \exp\left(-R_W^2/2\,\sigma^2\right) \right] \tag{2-30}$$

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2. The Probability of Coverage, Conditional Damage and Pro-
bability of Damage Functions for Multiply Delivered Unitary
Weapons

The substitution of R_W for R_p and R_L for R_{LB} in Equations
(2-3), (2-4) and (2-5) yield respectively, the probability of coverage,
conditional damage and probability of damage functions for m unitary
weapons distributed uniformly and at random over an area A_W delivered
against a point target.

$$P_C' = \left[1 - \exp\left(-R_W^2/2\,\sigma^2 \right) \right] \tag{2-31}$$

$$P_{D/C}' = \left[1 - \exp\left(-m\,R_L^2/R_W^2 \right) \right] \tag{2-32}$$

$$P_D' = \left[1 - \exp\left(-m\,R_L^2/R_W^2 \right) \right] \left[1 - \exp\left(-R_W^2/2\,\sigma^2 \right) \right] \tag{2-33}$$

where

$$\mu = m\,\pi\,R_L^2/\pi\,R_W^2 = m\,R_L^2/R_W^2$$

is the mean value over the pattern R_W.

3. Maximization of the Damage Functions

a. Multiply Delivered Area Munitions

$$\frac{\partial P_D'}{\partial R_W} = P_{D/C}'\,\frac{\partial P_C'}{\partial R_W} + P_C'\,\frac{\partial P_{D/C}'}{\partial R_p} = 0$$

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$$\frac{P_C'}{P_{D/C}'} = - \frac{\partial P_C'/\partial R_W}{\partial P_{D/C}'/\partial R_W}$$

operation on Equations (2-28) and (2-29) results in:

$$\frac{\left[1 - \exp\ (-R_W^{\ 2}/2\ \sigma^2)\right]}{\left[1 - \exp\ (-NR_p^{\ 2}\ P_{D/C}/R_W^{\ 2})\right]} = \frac{R_W^{\ 4}\left[\exp\ (-R_W^{\ 2}/2\ \sigma^2)\right]}{2NR_p^{\ 2}\ P_{D/C}\ \sigma^2\left[\exp(-NR_p^{\ 2}\ P_{D/C}/R_W^{\ 2})\right]}$$

which is satisfied when

$$R_W \doteq (2NR_p^{\ 2}\ P_{D/C}\ \sigma^2)^{1/4} \tag{2-34}$$

the radius which maximizes the damage function.

Substitution of (2-34) into (2-30) yields the damage function for the optimum pattern radius

$$P_D' = \left[1 - \exp(-R_p\ \sqrt{N\ P_{D/C}}\Big/\sqrt{2}\ \sigma)\right]^2 \tag{2-35}$$

and

$$N = (2/P_{D/C})\left[(\sigma/R_p)\ \ell n\ (1 - \sqrt{P_D'})\right]^2 \tag{2-36}$$

is the number of clusters, with individual pattern radii R_p and overall pattern radius R_W, required to achieve a specified level of damage P_D' when delivered with aiming error σ.

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b. Multiply Delivered Unitary Weapons

Similar procedures applied to Equations (2-31), (2-32) and (2-33) yield:

The pattern which maximizes the damage function,

$$R_W = (2m \, R_L^{\,2} \, \sigma^2)^{1/4} \tag{2-37}$$

The maximized damage function,

$$P_D' = \left[1 - \exp \left(- R_L \sqrt{m} \middle/ \sqrt{2} \, \sigma \right) \right]^2 \tag{2-38}$$

and,

$$m = 2 \left[(\sigma/R_L) \, \ell n \, (1 - \sqrt{P_D'}) \right]^2 \; ; \; (0 < P_D' < 1) \tag{2-39}$$

is the number of bombs necessary to achieve a level of damage P_D' when distributed over R_W and delivered with aiming error σ.

4. Optimum Cluster Weapons vs Optimum Unitary Weapons for Multiple Weapons Against a Point Target

Converting σ to CEP in Equations (2-35) and (2-38) and solving for CEP^2:

$$CEP^2 = \frac{N \, P_{D/C} \, R_D^{\,2} \, \ell n \, 2}{\left[\ell n \, (1 - \sqrt{P_D'}) \right]^2} \; ; \; (0 < P_D' < 1) \tag{2-40}$$

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$$CEP^2 = \frac{m \, R_L^2 \, \ell n \, 2}{\left[\ell n \, (1 - \sqrt{P_D'}) \right]^2} \; ; \; (0 < P_D' < 1) \qquad (2\text{-}41)$$

In Equation (2-40) there are N clusters each having weight $W_C = K_1 K_2 n$ as defined in paragraph (II-A-4-b).

$$NW_C = W_{CT}$$

or,

$$N = W_{CT}/W_C$$

Therefore,

$$CEP^2 = \frac{W_{uT} \, R_L^2 \, \ell n \, 2}{W_C \left[\ell n \, (1 - \sqrt{P_D'}) \right]^2} \; ; \; (0 < P_D' < 1) \qquad (2\text{-}42)$$

In Equation (2-41) there are m unitary weapons each having weight W_u. The total weight of the m unitary weapons is:

$$W_{uT} = m \, W_u$$

or

$$m = W_{uT}/W_u$$

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Therefore,

$$CEP^2 = \frac{W_{uT} \, R_L^2 \, \ell n \, 2}{W_u \left[\ell n \, (1 - \sqrt{P_D'}) \right]^2} \quad ; \quad (0 < P_D' < 1) \tag{2-43}$$

To compare the effectiveness of these two systems for equal aircraft loadouts in terms of total weight of munitions expended and equal delivery accuracy, equate (2-42) to (2-43) and solve for W_{CT} in terms of W_{uT}.

$$W_{CT} = \left(\frac{R_L^2}{R_p^2} \right) \left(\frac{W_C}{W_u} \right) \left(\frac{1}{P_{D/C}} \right) W_{uT} \quad ; \quad (0 < P_{D/C} \leq 1) \tag{2-44}$$

This relationship is independent of the damage probability as previously stated. It is assumed that both systems are delivered with the same accuracy. The following can be ascertained from Equation (2-44):

Clusters are preferred if:

$$\left(\frac{R_L^2}{R_p^2} \right) \left(\frac{W_C}{W_u} \right) \left(\frac{1}{P_{D/C}} \right) < 1 \quad ; \quad (0 < P_{D/C} \leq 1) \tag{2-45a}$$

Unitary weapons are preferred if:

$$\left(\frac{R_L^2}{R_p^2} \right) \left(\frac{W_C}{W_u} \right) \left(\frac{1}{P_{D/C}} \right) > 1 \quad ; \quad (0 < P_{D/C} \leq 1) \tag{2-45b}$$

where $P_{D/C}$ is determined by Equation (2-4).

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C. Single Massive Clusters versus Multiply Delivered Small Clusters

Many questions have arisen concerning the desirability of developing single massive clusters as opposed to large numbers of smaller clusters. These systems will be compared in the following paragraphs on an equal weight basis.

Let W_{MC} = Weight of the Massive Cluster

W_{CT} = Weight of N Small Clusters

The clusters are assumed to contain identical submunitions, the number in the massive cluster will be denoted as M and each small cluster will contain n bomblets.

The damage function for the massive cluster is:

$$P_D = \left[1 - \exp\left(- r_1 \, MR_{LB}^2 / R_W^2\right)\right]\left[1 - \exp\left(- R_W^2 / 2 \, \sigma^2\right)\right] \qquad (2\text{-}46)$$

and that of N smaller clusters is given by:

$$P_D = \left[1 - \exp\left(- NR_p^2 \, P_{D/C} / R_W^2\right)\right]\left[1 - \exp\left(- R_W^2 / 2 \, \sigma^2\right)\right] \qquad (2\text{-}47)$$

where $P_{D/C}$ is taken from (2-4), R_p is the pattern radius of a single small cluster, R_{LB} is the lethal radius of a single bomblet, and R_W is the radius of the overall munition pattern on the ground.

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Equation (2-46) is maximized when $R_W = (2MR_{LB}^2 \sigma^2)^{1/4}$

$$P_D = \left[1 - \exp\left(- R_{LB} \sqrt{r_1 M} \Big/ \sqrt{2} \, \sigma \right) \right]^2 \qquad (2\text{-}48)$$

and (2-47) has a maximum given by (2-35).

$$P_D = \left[1 - \exp\left(- R_p \sqrt{NP_{D/C}} \Big/ \sqrt{2} \, \sigma \right) \right]^2 \qquad (2\text{-}49)$$

In terms of CEP^2 (2-48) becomes

$$CEP^2 = \frac{r_1 \, W_{MC} \, R_{LB}^2 \, \ln 2}{K_1 K_2 \left[\ln\left(1 - \sqrt{P_D} \right) \right]^2}$$

and (2-49) becomes

$$CEP^2 = \frac{W_{CT} \, P_{D/C} \, R_p^2 \, \ln 2}{W_C \left[\ln\left(1 - \sqrt{P_D} \right) \right]^2}$$

For equal delivery accuracy and $W_C = K_1 K_2 n$

$$\frac{W_{CT} \, P_{D/C} \, R_p^2 \, \ln 2}{K_1 K_2 n \left[\ln\left(1 - \sqrt{P_D} \right) \right]^2} = \frac{r_1 \, W_{mC} \, R_{LB}^2 \, \ln 2}{K_1 K_2 \left[\ln\left(1 - \sqrt{P_D} \right) \right]^2}$$

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or,

$$W_{CT} = \left(\frac{r_1 n\ R_{LB}^2}{R_p^2\ P_{D/C}} \right) W_{mC}\ ;\ (0 < P_{D/C} \leq 1) \qquad (2\text{-}50)$$

The smaller clusters are preferred if:

$$\frac{r_1 n\ R_{LB}^2}{R_p^2\ P_{D/C}} < 1\ ;\ \left(0 < P_{D/C} \leq 1 \right)$$

and conversely.

SECTION III

OPTIMUM DAMAGE PROBABILITY AGAINST AREA TARGETS

A. Singly Delivered Weapons Against an Area Target

    1. The Probability of Fractional Coverage, Conditional Damage and Fractional Damage Functions for Area Weapons

        a. The Conditional Damage Function

        The conditional damage within the area of overlap between the target area and the pattern is identically the average probability of damage over the weapon pattern given in Equation (2-4) (Figure 6). It is repeated here for convenience.

$$P_{D/C} = \left[1 - \exp\left(- r_1 n \, R_{LB}^2 / R_p^2\right) \right] \tag{3-1}$$

        b. The Fractional Coverage Functions

        No single closed form approximation for the fractional coverage function could be found which yielded the desired accuracy over the entire R_T/σ range $(0 \leqslant R_T/\sigma \leqslant \infty)$. It was found that two expressions over appropriate intervals yielded acceptable accuracy over the applicable range of values.

        In the previous section, it was shown that the coverage function given in Equation (2-2) was accurate for all tactical surface targets where $R_T/\sigma \leqslant 0.5$ and $(0 \leqslant R_p/\sigma \leqslant \infty)$.

        For the range $(0.5 < R_T/\sigma < R_p/\sigma)$ the coverage function is more accurately approximated by:

$$F_C = C_1 \left[1 - \exp\left(- C_2 R_p^2 / \sigma^2\right) \right] \tag{3-2}$$

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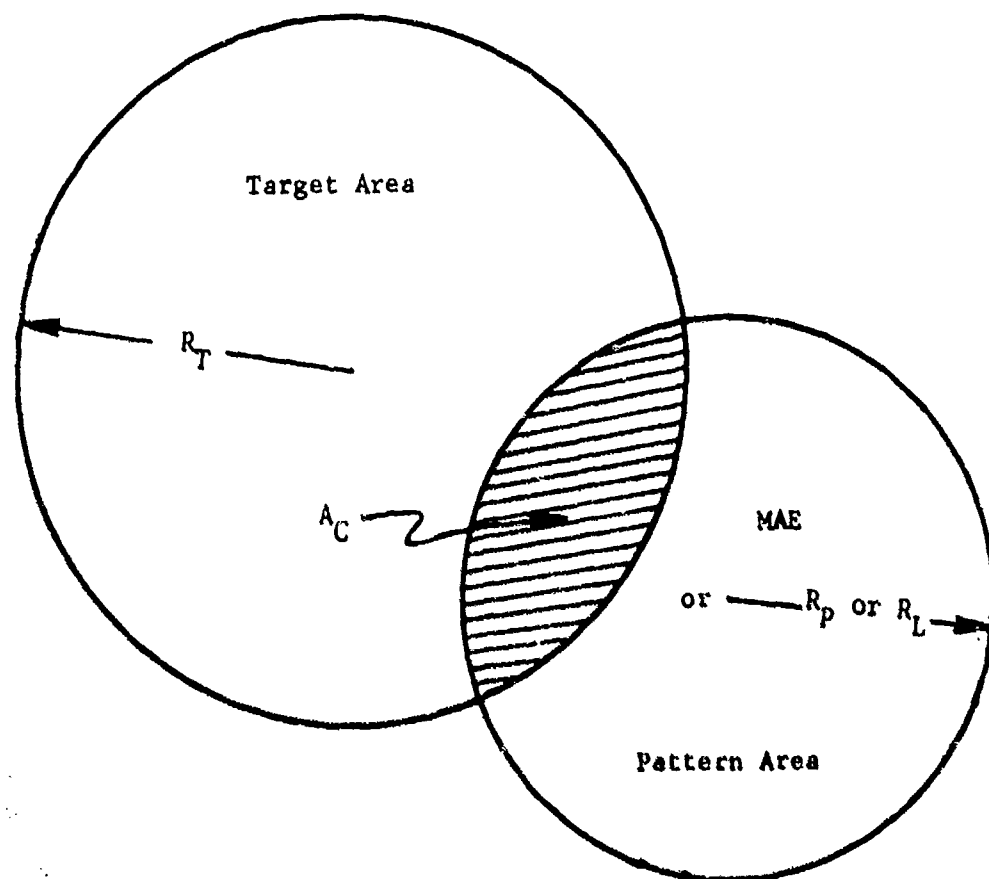
$$F_C = A_C/\pi R_T^2 = \text{Percentage of Target Area Covered}$$

Figure 6. Single Area or Single Unitary Weapon Cover Function

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Over the interval $(R_p/\sigma \leq R_T/\sigma \leq \infty)$, the coverage function is approximated by:

$$F_C = C_3 \frac{R_p^2}{R_T^2} \left[1 - \exp\left(- C_4 R_T^2/\sigma^2\right) \right] \qquad (3\text{-}3)$$

A sequential unconstrained minimization technique employing a non-gradient parameter search was used to determine the values of the coefficients C_i which minimized the error between the computed values of the coverage function and the numerically integrated values from reference 10. The initial error function utilized was:

$$ER = \sum_{i=1}^{n} \left| F_{CC_i} - F_{CT_i} \right|$$

where F_{CC_i} are the calculated values of the cover function and, F_{CT_i} are the numerically integrated values. The relative minimum program searched for values C_i which minimized this error function. The resulting coefficients while producing extremely accurate results over ninty-five percent of the range, produced unacceptable deviations for $R_T/\sigma \approx R_p/\sigma$. This error function was discarded in favor of minimizing the magnitude of the largest error, such that the maximum negative error matches the maximum positive error. For this fit the coefficients have the following values.

$C_1 = 1.0$ $\qquad\qquad\qquad\qquad$ $C_2 = 0.41$

$C_3 = 1.0$ $\qquad\qquad\qquad\qquad$ $C_4 = 0.436$

c. The Fractional Damage Functions

The fractional damage functions over the two intervals are given by the products of Equation (3-1) and (3-2) or (3-3), respectively.

$$F_D = \left[1 - \exp\left(-r_1 n\, R_{LB}^2 / R_p^2\right)\right]\left[1 - \exp\left(-.41\, R_p^2 / \sigma^2\right)\right] \quad (3\text{-}4a)$$

$$F_D = \left[1 - \exp\left(-r_1 n\, R_{LB}^2 / R_p^2\right)\right] \frac{R_p^2}{R_T^2}\left[1 - \exp\left(-.436\, R_T^2 / \sigma^2\right)\right]$$

$$(3\text{-}4b)$$

2. The Probability of Fractional Coverage, Conditional Damage and Fractional Damage Functions for Unitary Weapons

The conditional damage for unitary warheads is unity, a consequence of the manner in which the MAE is defined for any given damage or incapacitation criterion. The fractional damage is identically the fraction of the target covered.

$$F_D = \left[1 - \exp\left(-.41\, R_L^2 / \sigma^2\right)\right] \quad (3\text{-}5a)$$

$$F_D = \frac{R_L^2}{R_T^2}\left[1 - \exp\left(-.436\, R_T^2 / \sigma^2\right)\right] \quad (3\text{-}5b)$$

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3. Maximization of the Damage Functions for Area Targets

In a manner similar to the procedure in paragraph (II-A-4), the fractional damage functions may be maximized by determining the constrained optimum pattern radius. The fractional damage function

$$F_D = P_{D/C} \cdot F_C$$

is maximized when

$$\frac{F_C}{P_{D/C}} = - \frac{\partial F_C/\partial R_p}{\partial P_{D/C}/\partial R_p}$$

a. Singly Delivered Area Weapons

On the interval $(0.5 < R_T/\sigma < R_p/\sigma)$, this expression results in:

$$\frac{\left[1-\exp(-.41R_p^2/\sigma^2)\right]}{\left[1-\exp(-r_1 nR_{LB}^2/R_p^2)\right]} = \frac{.41\ R_p^4\left[\exp(-.41R_p^2/\sigma^2)\right]}{r_1 nR_{LB}^2 \sigma^2\left[\exp(-r_1 nR_{LB}^2/R_p^2)\right]}$$

which is satisfied when

$$R_p = (r_1 nR_{LB}^2\ \sigma^2/.41)^{1/4} \tag{3-6}$$

the pattern radius which maximizes the fractional damage. However, on the interval $(R_p/\sigma \leq R_R/\sigma \leq \infty)$ the result is:

$$\frac{\dfrac{R_p^2}{R_T^2}\left[1-\exp(-.436R_T^2/\sigma^2)\right]}{\left[1-\exp(-r_1 nR_{LB}^2/R_p^2)\right]} = \frac{R_p^4\left[\exp(-.436R_T^2/\sigma^2)\right]}{r_1 nR_{LB}^2\left[\exp(-r_1 nR_{LB}^2/R_p^2)\right]}$$

or

$$R_p^2 = \frac{r_1 nR_{LB}^2\left[\exp(-r_1 nR_{LB}^2/R_p^2)\right]}{\left[1-\exp(-r_1 nR_{LB}^2/R_p^2)\right]}$$

substituting $x = r_1 nR_{LB}^2/R_p^2$ yields after simplification:

$$x + 1 = \exp(x)$$

This expression is satisfied only at $x = 0$ and $x = \infty$. This implies that at $x = 0$, $R_p = \infty$ corresponding to a minimum conditional damage $(P_{D/C} \doteq 0.0)$ and a maximum coverage probability $(F_C \equiv 1.0)$. Also implicit is at $x = \infty$, $R_p = 0$ corresponding to a maximum conditional damage $(P_{D/C} \doteq 1.0)$ and a minimum coverage probability $(F_C \equiv 0.0)$. In both cases, r_1, n and R_{LB} are positive real numbers, and the damage probability is identically zero $(P_D \equiv 0.0)$.

The result is expected since by the imposed constraints $(R_p \leq R_T)$, at $R_p = \infty$, $R_T = \infty$. The two solutions form a coincident pair of global minima for the damage function, and being the only two solutions, existence of a maximum is precluded. Because of the constraints imposed

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as a consequence of the interval being considered $(R_p/\sigma \leq R_T/\sigma \leq \infty)$ the optimum damage function is not obtainable identically. That is, the condition $(P_{D/C} = F_C)$ is non-existent. Therefore, the best choice of pattern radius for any given munition/target/damage criterion combination must be determined numerically by iterating the pattern radius and evaluating the damage function.

This interval is academic insofar as conventional munitions design is concerned but is of considerable value for weapons effectiveness assessments purposes. Although conventional munitions cannot be designed specifically for vast area targets (due to their low yield), it is often the case that the damage potential of multiple sortie or multiple mission strikes must be predicted. If Equation (3-6) is substituted into (3-4a) the following expression is obtained:

$$F_D = \left[1-\exp\left(-\sqrt{.41r_1 n}\, R_{LB}/\sigma\right)\right]^2 \tag{3-7}$$

This is the maximum damage for the optimum pattern radius. Solving for the number of bomblets necessary to achieve a given fraction of damage for any specified target, damage criterion and aiming error, yields:

$$n = (1/.41r_1)\left[(\sigma/R_{LB})\, \ell n\,(1-\sqrt{F_D})\right]^2; \quad (0 < F_D < 1) \tag{3-8}$$

4. Optimum Cluster Weapons versus Optimum Unitary Weapons

Equations (3-7) and (3-5a) are converted to terms of CEP and solved for CEP^2.

Equation (3-7) becomes:

$$CEP^2 = \frac{.82 \; r_1 n R_{LB}^2 \, \ell n \, 2}{\left[\ell n \, (1- \sqrt{F_D}) \right]^2} \; ; \; (0 < F_D < 1) \tag{3-9}$$

or, in terms of weapon weight,

$$CEP^2 = \frac{.82 \; r_1 W_C R_{LB}^2 \, \ell n \, 2}{K_1 K_2 \left[\ell n \, (1- \sqrt{F_D}) \right]^2} \; ; \; (0 < F_D < 1) \tag{3-10}$$

Equation (3-5a) becomes:

$$CEP^2 = - \frac{.82 \; MAE \, \ell n \, 2}{\pi \, \ell n \, (1-F_D)} \; ; \; (0 < F_D < 1) \tag{3-11}$$

and in terms of weapon weight

$$CEP^2 = - \frac{.82 \; K \, W_u^\alpha \, \ell n \, 2}{\pi \, \ell n \, (1-F_D)} \; ; \; (0 < F_D < 1) \tag{3-12}$$

Equating (3-10) and (3-12) and solving for

$$W_C = - \frac{K_1 K_2 K}{r_1 \, \pi \, R_{LB}^2} \; \frac{\left[\ell n \, (1- \sqrt{F_D}) \right]^2}{\ell n \, (1-F_D)} \; W_u^\alpha \; ; \; (0 < F_D < 1) \tag{3-13}$$

which is identically (2-26) for $F_D = P_D$.

For equal system weights, this expression becomes:

$$W = - \left[\frac{K_1 K_2 K}{r_1 \, \pi \, R_{LB}^2} \; \frac{\left[\ell n \, (1- \sqrt{F_D}) \right]^2}{\ell n \, (1-F_D)} \right]^{\frac{1}{1-\alpha}} \; ; \; (0 < F_D < 1) \tag{3-14}$$

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By comparing Equations (2-27) and (3-14), it can be concluded that target area does not influence the choice of weapons systems for singly delivered weapons as long as the target consists of a distributed simple multiple of a point target. The term simple multiple refers to an area containing a number of identical or equally vulnerable point targets. An area containing a mixture of bare trucks and ordnance laden trucks as an example is excluded.

B. Multiply Delivered Weapons Against Area Targets

   1. The Probability of Fractional Coverage, Conditional Damage and Probability of Fractional Damage Functions for Area Weapons

      a. The Conditional Damage Function

      The conditional damage function for multiple area weapons was developed previously as reflected in Equation (2-29) and is repeated here for convenience.
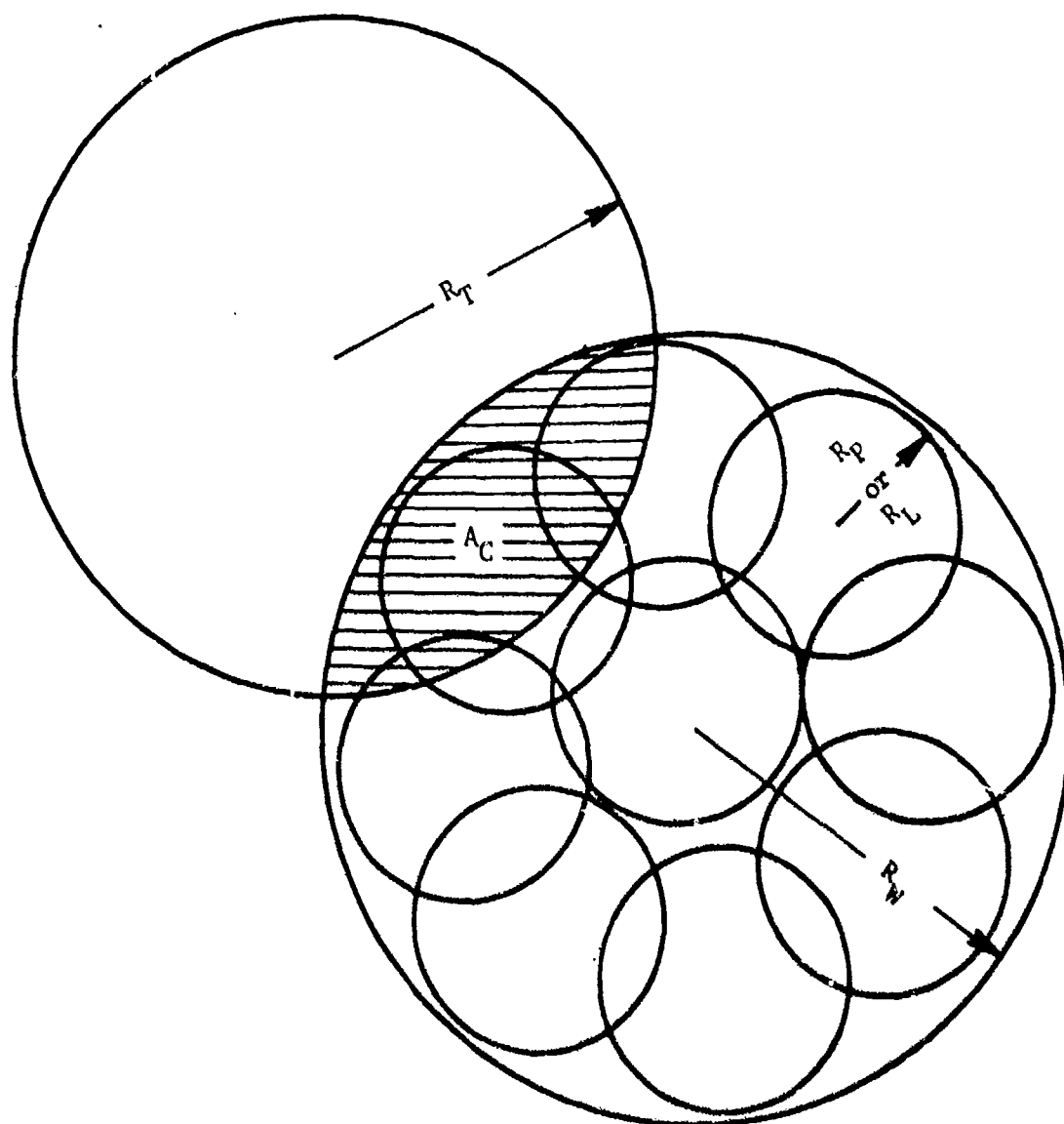
$$P_{D/C}' = \left[1-\exp(-NR_p^2 P_{D/C}/R_W^2)\right] \qquad (3-15)$$

      b. The Fractional Coverage Function

      Two different cases must be considered since the ratio R_W/σ may differ significantly from the ratio R_T/σ. For $R_T/\sigma \leq 0.5$ and the interval $(0 \leq R_W/\sigma \leq \infty)$ the coverage function is given by:

$$F_C' = \left[1-\exp(-R_W^2/2\sigma^2)\right] \qquad (3-16a)$$

The general case of fractional coverage for multiply delivered weapons is shown in Figure 7.

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$$F_C = A_C/\pi R_T^2$$

Figure 7. Multiple Area of Multiple Unitary Weapon Cover
Function

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and for the range $(0.5 < R_T/\sigma < R_W/\sigma$ it is approximated by:

$$F_C{}' = \left[1-\exp(-.41\ R_W{}^2/\sigma^2)\right] \qquad (3\text{-}16b)$$

For theoretical completeness, the interval $R_W/\sigma \le R_T/\sigma \le \infty$ must also be treated.

$$F_C{}' = \frac{R_W{}^2}{R_T{}^2}\left[1-\exp(-.436\ R_T{}^2/\sigma^2)\right] \qquad (3\text{-}17)$$

Again, the interval upon which (3-17) is based is of no practical value in _conventional_ weapons systems _design_, and is included only for theoretical completeness.

    c. The Fractional Damage Functions

        The probability of fractional damage is found by taking the products of Equations (3-15) and (3-16a), (3-16b) or (3-17) respectively:

$$F_D{}' = \left[1-\exp(-NR_p{}^2 P_{D/C}/R_W{}^2)\right]\left[1-\exp(-R_W{}^2/2\sigma^2)\right] \qquad (3\text{-}18a)$$

$$F_D{}' = \left[1-\exp(-NR_p{}^2 P_{D/C}/R_W{}^2)\right]\left[1-\exp(-.41R_W{}^2/2\sigma^2)\right] \qquad (3\text{-}18b)$$

$$F_D{}' = \left[1-\exp(-NR_p{}^2 P_{D/C}/R_W{}^2)\right]\ \frac{R_W{}^2}{R_T{}^2}\left[1-\exp(.436\ R_T{}^2/\sigma^2)\right] \qquad (3\text{-}18c)$$

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2. The Probability of Fractional Coverage, Conditional Damage
   and Probability of Fractional Damage Functions for Unitary
   Weapons

   a. The Conditional Damage Function

   The conditional damage function for multiply delivered
unitary weapons was developed earlier, in Equation (2-32), and is
repeated here for convenience:

$$P_{D/C}' = \left[1 - \exp(-mR_L^{\,2}/R_W^{\,2}) \right] \qquad (3-19)$$

   b. The Fractional Coverage Functions

   The fractional coverage functions for multiply delivered
weapons with pattern radius R_W are identical to those for multiply
delivered area weapons developed above. Equations (3-16a), (3-16b)
and (3-17) are utilized over the appropriate intervals.

   c. The Fractional Damage Functions

   Taking the appropriate products, the fractional damage
functions become:

$$F_D' = \left[1 - \exp(-mR_L^{\,2}/R_W^{\,2}) \right] \left[1 - \exp(-R_W^{\,2}/2\sigma^2) \right] \qquad (3-20a)$$

$$F_D' = \left[1 - \exp(-mR_L^{\,2}/R_W^{\,2}) \right] \left[1 - \exp(-.41\, R_W^{\,2}/2\sigma^2) \right] \qquad (3-20b)$$

$$F_D' = \left[1 - \exp(-mR_L^{\,2}/R_W^{\,2}) \right] \frac{R_W^{\,2}}{R_T^{\,2}} \left[1 - \exp(-.436\, R_T^{\,2}/\sigma^2) \right] \qquad (3-20c)$$

3. Maximization of the Damage Function

It can be shown in the manner described in the previous chapter that the damage functions are approximated by:

$$\frac{F_C^{'}}{P_{D/C}^{'}} = - \frac{\partial F_C^{'}/\partial R_W}{\partial P_{D/C}^{'}/\partial R_W}$$

a. Multiply Delivered Area Munitions

Operation on Equations (3-15) and (3-16a) leads to Equations (3-15) and (3-16a) leads to:

$$\frac{\left[1-\exp(-.41\ R_W^{\,2}/\sigma^2)\right]}{\left[1-\exp(-NR_p^{\,2}P_{D/C}/R_W^{\,2})\right]} = \frac{.41\ R_W^{\,4}\left[\exp(-.41\ R_W^{\,2}/\sigma^2)\right]}{NR_p^{\,2}P_{D/C}\ \sigma^2\left[\exp(-NR_p^{\,2}P_{D/C}/R_W^{\,2})\right]}$$

This relation is satisfied when:

$$R_W = (NR_p^{\,2}P_{D/C}\ \sigma^2/.41)^{1/4} \tag{3-21}$$

the pattern radius which maximizes the fractional damage.

Substituting Equation (3-21) into (3-18b) the following expression results:

$$F_D^{'} = \left[1-\exp(-\sqrt{.41\ NP_{D/C}}\ R_p/\sigma)\right]^2 \tag{3-22}$$

This represents the maximum damage commensurate with the optimum pattern

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radius. The number of clusters required to achieve a specified fraction of damage is given by:

$$N = (1/.41 \, P_{D/C})\left[(\sigma/R_p) \, \ell n \, (1- \sqrt{F_D'})\right]^2 \; ; \; (0 < F_D' < 1) \qquad (3\text{-}23)$$

b. Multiply Delivered Unitary Munitions

Operation on Equations (3-19) and (3-16a) leads to Equations (3-19) and (3-16b) leads to:

$$\frac{\left[1-\exp(-.41 \, R_W^2/\sigma^2)\right]}{\left[1-\exp(-mR_L^2/R_W^2)\right]} = \frac{.41 \, R_W^4\left[\exp(-.41 \, R_W^2/\sigma^2)\right]}{2mR_L^2\sigma^2\left[\exp(-mR_L^2/R_W^2)\right]}$$

This relation is satisfied when:

$$R_W = (mR_L^2\sigma^2/.41)^{1/4} \qquad (3\text{-}24)$$

This is the pattern radius which maximizes the fractional damage. The maximum fractional damage is obtained by substituting this expression into Equation (3-20b) yielding:

$$F_D' = \left[1-\exp(- \sqrt{.41m} \; R_L/\sigma)\right]^2 \qquad (3\text{-}25)$$

The number of unitary weapons of lethal radius R_L required to achieve a specified fraction of damage F_D' against an area target of radius R_T when delivered with aiming error σ is found by solving the above expression for:

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$$m = (1/.41)\left[(\sigma/R_L)\, \ell n \, (1-\sqrt{F_D{}'})\right]^2 \; ; \; (0 < F_D{}' < 1) \qquad (3\text{-}26)$$

4. Optimum Cluster Weapons versus Optimum Unitary Weapons

Equations (3-22) and (3-25) are converted to terms of CEP and solved for CEP^2. Equation (3-22) becomes:

$$CEP^2 = \frac{.82 \; N \; P_{D/C} \; R_p{}^2 \; \ell n \, 2}{\left[\ell n \, (1-\sqrt{F_D{}'})\right]^2} \; ; \; (0 < F_D{}' < 1) \qquad (3\text{-}27)$$

And in terms of weapon weight:

$$CEP^2 = \frac{.82 \; W_{CT} P_{D/C} R_p{}^2 \; \ell n \, 2}{W_C\left[\ell n \, (1-\sqrt{F_D{}'})\right]^2} \; ; \; (0 < F_D{}' < 1) \qquad (3\text{-}28)$$

Equation (3-25) becomes:

$$CEP^2 = \frac{.82 \; mR_L{}^2 \; \ell n \, 2}{\left[\ell n \, (1-\sqrt{F_D{}'})\right]^2} \; ; \; (0 < F_D{}' < 1) \qquad (3\text{-}29)$$

And in terms of weapon weight:

$$CEP^2 = \frac{.82 \; W_{UT} R_L{}^2 \; \ell n \, 2}{W_u\left[\ell n \, (1-\sqrt{F_D{}'})\right]^2} \; ; \; (0 < F_D{}' < 1) \qquad (3\text{-}30)$$

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Equating (3-28) and (3-30) and solving for W_{CT}:

$$W_{CT} = \left(\frac{R_L^2}{R_p^2}\right)\left(\frac{W_C}{W_u}\right)\left(\frac{1}{P_{D/C}}\right) \quad W_{UT} \; ; \; (0 < P_{D/C} \le 1) \tag{3-31}$$

This expression permits comparisons of the two weapons systems on an equal weight basis. The expression is independent of the fractional damage, inferring that it holds for any specified level of damage, providing the respective systems being analyzed are capable of achieving the specified level of damage. However, the weight requirements for high fractions of damage against area targets may be prohibitively large.

It is seen from Equation (3-31) that clusters are preferred if:

$$\left(\frac{R_L^2}{R_p^2}\right)\left(\frac{W_C}{W_u}\right)\left(\frac{1}{P_{D/C}}\right) \; < 1 \; ; \; (0 < P_{D/C} \le 1)$$

and, unitary weapons are preferred if:

$$\left(\frac{R_L^2}{R_p^2}\right)\left(\frac{W_C}{W_u}\right)\left(\frac{1}{P_{D/C}}\right) \; > 1 \; ; \; (0 < P_{D/C} < 1)$$

It is significant to note that Equation (3-31) is identically (2-44) and it can be concluded that _target area does not influence the choice of weapons systems for multiply delivered weapons._

This conclusion, with the conclusion reached in the preceeding section on singly delivered weapons against area targets leads to the general conclusion that:

The choice of weapon system for any specified target is independent of the size of the target as long as the target consists of a distributed simple multiple of a point target.

This conclusion has a far ranging impact on current weapon system effectiveness analyses. Approximately 50-70 percent of all analysis efforts are devoted to analyzing the effects of target area (area targets consisting of uniformly distributed point targets) on weapon system preference. These studies have been shown to be redundant. Elimination of these studies will eliminate a major workload in weapon system effectiveness analysis.

SECTION IV

OPTIMUM DAMAGE PROBABILITY AGAINST TARGETS

WITH LOCATION UNCERTAINTY

Independent mathematical derivations involving a point target located at random within a specified area, where the actual location uncertainty has some probability distribution about the area centroid, lead to mathematical relationships equivalent to those in section III.

The dynamic situation occurs when aircraft on search and destroy missions, acquire a mobile target at some fixed ground coordinate, but are not in position to deliver weapons against the target immediately. During the relatively short time required for the aircraft to convert to an offensive strike attitude, the target has the latitude to maneuver or seek cover of the local terrain and vegetation. Although the pilot may not reacquire the target specifically, due to possible masking which has occurred during the lapsed time interval, there is a realistic probability of damage associated with the delivery of weapons against the original acquisition coordinates providing the lapsed time increment is small compared to the target's evasive capability.

An analogous situation is a mobile target moving through a treeline yielding fleeting positions to the attack aircraft. The motion and exact location are distorted, but a relative area of location is identifiable.

It became apparent early in the development of the mathematics of this section that this dynamic situation was equivalent to the

static situation of the previous section. The conclusion reached is that:

The probability of damaging a point target within an area of radius R_W, where the location of the target is defined by standard deviation σ_1, the aircraft delivery error is defined by standard deviation σ_A, and the total error σ_T is a convolution of the two former distrubutions, is identically the probability of fractional damage against an area target of radius R_W attacked with aiming error σ_T. Bryant's [9] work adequately treats this problem and is recommended to readers wishing to explore this subject further.

In the case of similar distributions in target location and aiming error, the variance of the total error distribution is the sum of the variances of the individual distributions. The mathematics of this section are in this respect redundant and are excluded. The results of analyses involving area targets are interpreted with regard to this analogy.

APPENDIX

ANNULAR PATTERN METHODOLOGY

The coverage function for annular patterns may be determined by considering the coverage probabilities of the associated outer (R_o) and inner (R_1) radii. The probability that the target lies within the area generated by R_o is:

$$P_{Co} = \left[1.-\exp(-R_o^2/2\sigma^2)\right] \tag{A-1}$$

and the probability that it lies within the area generated by the R_1 is:

$$P_{C1} = \left[1.-\exp(-R_1^2/2\sigma^2)\right] \tag{A-2}$$

Finally, the probability that the target lies in the annular ring is the difference in (A-1) and (A-2);

$$P_C = \left[\exp(-R_1^2/2\sigma^2) - \exp(-R_o^2/2\sigma^2)\right] \tag{A-3}$$

The conditional damage function must also be modified since the lethal effects are confined to the annular ring. The mean value becomes:

$$\mu = r_1 nMAE_B/(A_o-A_1)$$

or

$$\mu = r_1 nR_{LB}^2/(R_o^2 - R_1^2)$$

and the conditional damage function is given by:

$$P_{D/C} = \left[1 - \exp(-r_1 nR_{LB}^2/(R_o^2 - R_1^2))\right] \qquad (A-4)$$

and finally, the damage probability is given by:

$$P_D = \left[\exp(-R_1^2/2\sigma^2) - \exp(-R_o^2/2\sigma^2)\right] \quad X$$

$$\left[1 - \exp(r_1 nR_{LB}^2/(R_o^2 - R_1^2))\right] \qquad (A-5)$$

For multiple weapons employment, the area of the resulting central void must be determined and the coverage function modified in a manner similar to the above. The conditional damage function is found simply by replacing R_p^2 in Equation (2-29) with $(R_o^2 - R_1^2)$ and making the appropriate substitutions for R_W^2.

$$P_{D/C}' = \left\{1 - \exp\left[-N(R_o^2 - R_1^2) \, P_{D/C}/R_W^{*2}\right]\right\}$$

where R_W^* is a function of the difference between the overall pattern area and the resulting central void if it exists.

Appropriate combinations of the above equations will yield the damage equations for area targets as demonstrated in Section III.

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*	2a. REPORT SECURITY CLASSIFICATION
Plans Office	Unclassified
Air Force Armament Laboratory	2b. GROUP
Eglin Air Force Base, Florida 32542	n/a

3. REPORT TITLE

AIR ARMAMENT PLANNING AND DESIGN THROUGH SYSTEMS ANALYSIS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Final Report (December 1970 - December 1971)

5. AUTHOR(S) *(First name, middle initial, last name)*

John H. Arnold

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
February 1972	81	10

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	AFATL-TR-72-28
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

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Distribution limited to U. S. Government agencies only; this report documents test and evaluation; distribution limitation applied February 1972 . Other requests for this document must be referred to the Air Force Armament Laboratory (DLX), Eglin Air Force Base, Florida 32542.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
Available in DDC	Air Force Armament Laboratory
	Air Force Systems Command
	Eglin Air Force Base, Florida 32542

13. ABSTRACT

Closed-form approximations for the probability of damaging surface targets with aerially delivered weapons are developed and analyzed for six different employment situations; single weapons against point targets, multiple weapons against point targets, single weapons against area targets, multiple weapons against area targets, single weapons against point targets with location uncertainty, and multiple weapons against point targets with location uncertainty. In each case, conditional damage and probability of coverage functions are developed, the product of which defines the probability of damage or probability of fractional damage depending upon whether the target is a point or an area, respectively. In addition, optimum damage probability constrained by specific design characteristics and delivery errors, is developed and compared with the capabilities of current systems. Optimum pattern radii or pattern radii which maximize the damage probability are also developed. Methodology which leads to preliminary design characteristics is developed through determination of the number of submunitions or weapon weight required to achieve any given level of damage for given employment constraints. Weapon preference methodology is developed which establishes a parametric evaluation procedure for weapon system employment preference and preliminary design characteristics. The methodology relates specifically to continuous patterns, that is, to weapons impact patterns bounded by singly connected curves and containing a random distribution of submunitions over the patterns. The principles are also extended to weapons systems with impact patterns that are annular in nature

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Air Armament Planning						
Systems Analysis						
Surface Targets						
Aerially Devliered Weapons						
Area Targets						
Point Targets						
Annular Pattern Methodology						

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either circular or elliptic (within established limits), and that are bound by
multiply connected outer and inner curves. For this application, the submunitions
are constrained to lie within the annular ring or the area between the outer and
inner curves. The methodology is accurate and requires very little manpower and
computer resources to employ. It is based on the mean area of effectiveness
concept and can readily and accurately be used to assess the potential of new
designs and proposals if accurate estimates of the mean area of effectiveness
can be made from the lethal performance of existing munitions and submunitions.